Numeric Response Questions

Limits

Q.1 Find the value of $\lim_{x\to 0}\left(\left[\frac{100x}{\sin x}\right]+\left[\frac{99\sin x}{x}\right]\right)$

(Where $[.]$ *represents the greatest integral function*).

Q.2 Evaluate: $\lim_{n\to\infty}\left[\frac{1}{n^3+1}+\frac{4}{n^3+1}+\frac{9}{n^3+1}+\cdots+\frac{n^2}{n^3+1}\right]$.

Q.3 If : $\lim_{x\to 0}\frac{8}{x^8}\left[1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{2}\cos\frac{x^2}{4}\right]$ is equal to $\frac{\lambda}{k}$ then find $k-\lambda$.

Q.4 Evaluate: $\lim_{x\to\frac{\pi}{6}}\frac{2\sin^2 x+\sin x-1}{2\sin^2 x-3\sin x+1}$,

Q.5 Evaluate : $\lim_{n\to\infty}(3^n+4^n)^{1/n}$

Q.6 If $\lim_{x\to 0}\frac{1-\cos x}{x(2^x-1)}=k\log z\ e$ then find k.

Q.7 Evaluate : $\lim_{x\to 0}\left[\left(\min^m(y^2-4y+11)\right)\frac{\sin x}{x}\right]$ (where $[.]$ represent greatest integer function).

Q.8 If $f(x)=\frac{\sin(e^{x-2}-1)}{\log(x-1)}$, then find $\lim_{x\to 2}f(x)$,

Q.9 Evaluate: $\lim_{x\to 0}\frac{\sin 4x}{1-\sqrt{(1-x)}}$.

Q.10 If $\lim_{x\to 0}\frac{(1+x)^{1/x}-e+\frac{1}{2}ex}{x^2}=\frac{11e}{k}$ then find k.

Q.11 Let $f(n)=\lim_{x\to 0}\left(\left(1+\sin\frac{x}{2}\right)\left(1+\sin\frac{x}{2^2}\right)\ldots\left(1+\sin\frac{x}{2^n}\right)\right)^{1/x}$. If $\lim_{n\to\infty}f(n)=\frac{3e}{k}$ then find k.

Q.12 If $\lim_{n\to\infty}\left[\frac{1}{3.7}+\frac{1}{7.11}+\frac{1}{11.15}+\cdots\ldots+\frac{1}{(4n-1)(4n+3)}\right]=\frac{2}{k}$ then find k.

Q.13 Let $f(x)=3x^{10}-7x^4+5x^4-21x^3+3x^2-7$. Then find the value of $\lim_{h\to 0}\frac{f(1-h)-f(1)}{h^3+3h}$,

Q.14 Evaluate: $\lim_{x\to 1}\frac{1+\cos\pi x}{\tan^2\pi x}$

Q.15 Evaluate: $\lim_{x\to 0}\left(\frac{e^x-e^{\sin x}}{x-\sin x}\right)$

ANSWER KEY

1. 198.00	**2.** 0.33	**3.** 31.00	**4.** –3.00	**5.** 4.00	**6.** 0.50	**7.** 6.00
8. 1.00	**9.** 8.00	**10.** 24.00	**11.** 3.00	**12.** 24.00	**13.** 17.66	**14.** 0.50
15. 1.00						

Hints & Solutions

1. By result

$$\lim_{x\to 0}\left[a\frac{\sin x}{x}\right] = a - 1$$

$$\lim_{x\to 0}\left[\frac{100x}{\sin x}\right]+\left[99\frac{\sin x}{x}\right] = 100 + 98 = 198$$

2. $$\lim_{n\to\infty}\left[\frac{1}{n^3+1}+\frac{4}{n^3+1}+\frac{9}{n^3+1}+.....\frac{n^2}{n^3+1}\right]$$

$$= \lim_{n\to\infty}\frac{n(n+1)(2n+1)}{6(n^3+1)} = \frac{1}{3}$$

3. $$\lim_{x\to 0}\frac{8}{x^8}\left(1\left(1-\cos\frac{x^2}{2}\right)-\cos\frac{x^2}{4}\left(1-\cos\frac{x^2}{2}\right)\right)$$

$$\lim_{x\to 0} 8\frac{\left(1-\cos\frac{x^2}{2}\right)\left(1-\cos\frac{x^2}{4}\right)}{x^8}$$

$$\lim_{x\to 0} 8\frac{2\sin^2\frac{x^2}{4}.2\sin^2\frac{x^2}{8}}{x^8}$$

$$\lim_{x\to 0} 8\frac{2\left(\frac{\sin\frac{x^2}{4}}{\frac{x^2}{4}}\right)^2.2\left(\frac{\sin\frac{x^2}{8}}{\frac{x^2}{8}}\right)^2.\left(\frac{x^2}{4}\right)^2\left(\frac{x^2}{8}\right)^2}{x^8}$$

$$= 8.\frac{1}{256} = \frac{1}{32}$$

4. $$\lim_{x\to\pi/6}\frac{(2\sin x-1)(\sin x+1)}{(2\sin x-1)(\sin x-1)}$$

$$= \lim_{x\to\pi/6}\frac{\sin x+1}{\sin x-1} = -3$$

5. $$\lim_{n\to\infty} 4\left(1+\left(\frac{3}{4}\right)^n\right)^{1/n} = 4$$

6. $$\lim_{x\to 0}\frac{1-\cos x}{x(2^x-1)}.\frac{x}{e^x-1}$$

$$= \lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{4\left(\frac{x}{2}\right)^2}.\frac{x}{x\log_e 2-1}$$

$$= \lim_{x\to 0}\frac{1}{2}\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2 \times \frac{x}{(1+x\log_e 2+....)-1}$$

$$= \frac{1}{2}\log_2 e$$

7. $$\lim_{x\to 0}\left[(\min{}^m(y^2-4y+11))\frac{\sin x}{x}\right]$$

$$= \lim_{x\to 0}\left[\frac{7\sin x}{x}\right] = 6$$

9. $$\lim_{x\to 0}\frac{\sin 4x}{1-\sqrt{(1-x)}}$$

$$= \lim_{x\to 0}\frac{\sin 4x}{1-\sqrt{1-x}}\times\frac{1+\sqrt{1-x}}{1+\sqrt{1-x}}$$

$$= \lim_{x\to 0}\frac{\sin 4x}{1-(1-x)}\times(1+\sqrt{1-x})$$

$$= \lim_{x\to 0} 4\times\frac{\sin 4x}{4\times x}\times(1+\sqrt{1-x})$$

$$4 \times (1 + 1) = 4 \times 2 = 8$$

10. Using expansion

$$\lim_{x\to 0}\frac{\left(e-\frac{ex}{2}+\frac{11e}{24}x^2.......\right)-e+\frac{ex}{2}}{x^2}=\frac{11e}{24}$$

11. $f(n) = \lim_{x\to 0}\left(1+\sin\frac{x}{2}\right)^{1/x}\left(1+\sin\frac{x}{2^2}\right)^{1/x}$

$.........\left(1+\sin\frac{x}{2^n}\right)^{1/x}$

$= e^{\frac{1}{2}}.\ e^{\frac{1}{2^2}}\ ....\ e^{\frac{1}{2^n}}$

Now, $\lim_{n\to\infty} f(n) = e^{\frac{1/2}{1-1/2}} = e$

12. $\lim_{n\to\infty}\frac{1}{4}$

$$\left[\frac{7-3}{3.7}+\frac{11-7}{7.11}+\frac{15-11}{11.15}+....+\frac{(4n+3)-(4n-1)}{(4n-1).(4n+3)}\right]$$

$= \lim_{n\to\infty}\frac{1}{4}$

$$\left[\left(\frac{1}{3}-\frac{1}{7}\right)+\left(\frac{1}{7}-\frac{1}{11}\right)+\left(\frac{1}{11}-\frac{1}{15}\right)+....\ ...+\left(\frac{1}{4n-1}-\frac{1}{4n+3}\right)\right]$$

$= \lim_{n\to\infty}\frac{1}{4}\left[\frac{1}{3}-\frac{1}{4n+3}\right]=\frac{1}{12}$

13. L Hospital rule:

given limit is $f'(1)\left(-\frac{1}{3}\right)=\frac{53}{3}$

$f'(x) = 30x^9 - 56x^7 + 30x^5 - 63x^2 + 6x$

$f'(1) = -53$

So, $-\frac{f'(1)}{3}=\frac{53}{3}$

14. $\lim_{x\to 1}\frac{(1+\cos\pi x)}{\tan^2\pi x}\ \left(\frac{0}{0}\text{ form}\right)$

Using L'Hospital's rule

$\lim_{x\to 1}\frac{-\pi\sin\pi x}{2\pi\tan\pi x\ \sec^2\pi x}$

$= \lim_{x\to 1}-\frac{1}{2}\cos^3\pi x = \frac{1}{2}$

15. $\lim_{x\to 0} e^x\left(\frac{e^{\sin x - x}-1}{\sin x - x}\right) = e^0 \times 1 = 1$